

ESTIMATE OF MULTIPLIERS FOR THE PUERTO RICAN ECONOMY

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Multiplier Analysis

Repercussions of changes in the level of expenditures on total output, income and employment can be estimated via the concept of the multiplier.

The formal analysis of the concept is associated with the names of R.F.Kahn and J.M. Keynes. However implicit versions of this concept are found in F. Quesnay writings. In his book "Tableau Economique", describing the operation of a farm, Quesnay showed the successive rounds of productive activities in response to a given increase in production¹.

The Keynesian income multiplier developed in macroeconomic theory is an aggregate multiplier which fails to distinguish between the sectors in which the initial expenditure changes originate. It has direct analogies at the regional level in the form of regional income and economic base multipliers. Input-output models, on the other hand enable us to derive sets of multipliers the main feature of which is that they are disaggregated, recognizing that the total impact on income (output, employment) will vary according to which sector experiences the initial expenditure change. Manipulation of the Input-output table allows the analyst to estimate different types of multiplier depending on whether he is interested in output, income or employment effects. Also, the values attached to the income and employment multipliers are not uniquely determined but are governed by the degree of model closure (i.e. the allocation of sectors between the endogenous matrix and final demand). The objectives of this work are to show how alternative multipliers may be derived.

¹ J. M. Keynes, *The Means to Prosperity*, Londres (1933). R. F. Kahn, "The Relation of Home Investment to Unemployment", *Economic Journal* (June 1931) pp 173-198.

W. H. Miernyk, *The Elements of Input-Output Analysis*, Random House, Nueva York (1965) page 4.

There are three Input-Output multipliers: output, employment and income multipliers. Income and employment multipliers can be classified as type 1 and type 2. Type one is estimated using the open input-output model. Type 2 multipliers are estimated closing the model to include the household sector in the endogenous matrix.

Output (or column) multiplier

The output multiplier measures the sum of direct and indirect output requirements from all sectors needed to deliver one additional dollar of output of *i* industry to final demand. It is derived by summing the entries in the column under industry *i* in the Leontief inverse matrix tables. Although the output multiplier represents total requirements per unit of final output, it is not a particularly useful concept except as an indicator of the degree of structural interdependence between each sector and the rest of the economy. In economic impact studies we are more usually concerned with income or employment generating effects, and these require income or employment multipliers. Table 1 below shows the domestic inverse matrix for year 1992. In the last row of the table the numbers under each of the eight industries represent the output multiplier.

TABLE 1
DOMESTIC INVERSE MATRIX 8*8 SECTORS, PUERTO RICO, 1992

Industrial Sector	Agriculture	Mining and Construction	Manufacture	Transportation Communications And Public Services	Trade	Finance Insurance and Real Estate	Services	Government
Agriculture	1.0173	0.0069	0.0163	0.0024	0.0049	0.0031	0.0022	0.0036
Mining and Construction	0.0069	1.0122	0.0115	0.0104	0.0216	0.0309	0.0113	0.0098
Manufacture	0.1799	0.2175	1.1651	0.1123	0.0567	0.0480	0.0539	0.0634
Transportation Communication and Other Public Services	0.0244	0.0544	0.0441	1.1221	0.0788	0.0661	0.0553	0.0484
Trade	0.0743	0.1146	0.0687	0.0404	1.0674	0.0593	0.0395	0.0443
Finance, Insurance and Real Estate	0.0240	0.0738	0.0425	0.0687	0.1038	1.1972	0.0661	0.0900
Services	0.0310	0.1342	0.0618	0.1297	0.0930	0.1215	1.0965	0.0914
Government	0.0016	0.0064	0.0030	0.0181	0.0058	0.0192	0.0218	1.0121
Output Multiplier	1.3592	1.6199	1.4129	1.5041	1.4320	1.5453	1.3468	1.3629

The output multiplier of manufacturing sector is 1.4129, which means that that this sector and the rest of the system have to produce \$1.43 in order to deliver a dollar of manufacturing final demand.

Income multiplier (Type I)

This is expressed as the ratio of the direct plus the indirect income change to the direct income change resulting from a unit increase in final demand for any given sector. In the case of Puerto Rico, the direct income change for each sector is given by the wage and salary row entry of the value added matrix when expressed in input coefficient form (i.e. the direct coefficients table). In other words, salaries and wages for each industry are divided by the corresponding output. The last row of Table 2 shows the direct income coefficients.

**TABLE 2
DIRECT INCOME COEFFICIENTS, PUERTO RICO, INPUT-OUTPUT TABLE 1992**

Industrial Sector	Wages and salaries	Output	Direct Income Coefficient
Agriculture	172,411	701,575	0.2457
Mining and Construction	552,159	3,357,165	0.1645
Manufacture	2,770,994	31,748,442	0.0873
Transportation Communication and Other Public Services	1,036,098	5,057,232	0.2049
Trade	1,874,190	9,079,514	0.2064
Finance, Insurance and Real Estate Services	850,212	8,177,154	0.1040
Government	1,954,786	7,671,280	0.2548
	3,587,726	6,292,772	0.5701
Total	12,798,576	72,085,134	0.1775

The direct and indirect income coefficients are obtained by multiplying the direct coefficient row vector **W** in the equation below) by the standard inverse matrix (i.e. households excluded). Thus, the direct and indirect income change for sector j is given by

$$Y_1 = W(I-A)^{-1}$$

where

W is the row vector of income coefficients, $(I-A)^{-1}$ is the domestic inverse and Y_1 is the vector of direct and indirect income coefficients.

The the multiplier type 1 is the ratio of the direct and indirect income change to the direct income change due to a unit increase in final demand. Table 3 shows the income multiplier for eight sectors.

TABLA 3
INCOME MULTIPLIER TYPE 1, PUERTO RICO, 1992

Industrial Sector	Direct Income Coefficient	Direct Plus Indirect Income Coefficient	Multiplier Type I
Agriculture	0.2457	0.2984	1.214
Mining and Construction	0.1645	0.2675	1.626
Manufacture	0.0873	0.1526	1.749
Transportation Communication and Other Public Services	0.2049	0.3008	1.468
Trade	0.2064	0.2840	1.376
Finance, Insurance and Real Estate	0.1040	0.2022	1.945
Services	0.2548	0.3253	1.277
Government	0.5701	0.6367	1.117

Income multiplier (Type II)

This is the ratio of the direct, indirect and induced income change to the direct income change due to a unit increase in final demand. The Type II multiplier takes into account the repercussionary effects of secondary rounds of consumer spending in addition to the direct and indirect interindustry effects. Income expansion due to successive 'rounds' of consumer spending is derived by expanding the interindustry matrix (A) by inclusion of the household row and column thereby making the household sector endogenous. The direct, indirect and induced income change per unit of final demand is shown by the household coefficient in the table of direct and indirect requirements derived from the expanded matrix with households endogenous; the household row in this inverted matrix lists the direct indirect and induced coefficients for each sector. The direct income change is exactly the same as in the preceding case. In Puerto Rico the household sector consists of a row of wages and salaries and column vector of consumption. Table 4 shows the augmented inverse. Direct, indirect and induced coefficients are shown in the last row of the matrix.

TABLE 4
AUGMENTED DOMESTIC INVERSE (9*9), PUERTO RICO, 1992

Industrial Sector	Agriculture	Mining and Construction	Manufacture	Transportation Communications And Public Services	Trade	Finance Insurance and Real Estate	Services	Government	Households
Agriculture	1.0228	0.0119	0.0192	0.0080	0.0102	0.0069	0.0083	0.0155	0.0187
Mining and Construction	0.0143	1.0189	0.0153	0.0179	0.0287	0.0359	0.0195	0.0257	0.0250
Manufacture	0.2893	0.3155	1.2211	0.2226	0.1608	0.1221	0.1732	0.2968	0.3665
Transportation Communication and Other Public Services	0.0864	0.1100	0.0757	1.1845	0.1378	0.1080	0.1229	0.1805	0.2075
Trade	0.2126	0.2385	0.1394	0.1799	1.1989	0.1530	0.1903	0.3394	0.4634
Finance, Insurance and Real Estate	0.1567	0.1927	0.1104	0.2025	0.2302	1.2872	0.2108	0.3733	0.4449
Services	0.1410	0.2327	0.1180	0.2405	0.1977	0.1960	1.2164	0.3260	0.3685
Government	0.0164	0.0197	0.0106	0.0331	0.0199	0.0293	0.0380	1.0438	0.0498
Households	0.4021	0.3604	0.2057	0.4054	0.3826	0.2725	0.4384	0.8580	1.3474

The direct, indirect and induced coefficients can also be estimated by using the following equation.

$$Y_2 = W(I - \bar{A})^{-1}$$

where

Y_2 is the vector of direct, indirect, and induced coefficients, W is the vector of direct coefficients and $(I - \bar{A})^{-1}$ is the augmented inverse. The ratio of direct, indirect and induced coefficients to the direct coefficients is known as the income multiplier type 2 or,

$$M_2 = \frac{Y_2}{W}$$

where M_2 is the vector of multipliers. Table 5 shows the income multiplier type 2.

TABLE 5
INCOME MULTIPLIERS TYPE 2, PUERTO RICO 1992

Industrial Sector	Direct Income Coefficients	Direct, Indirect and Induced Income Coefficients	Income Multiplier Type 2
Agriculture	0.2457	0.4021	1.636
Mining and Construction	0.1645	0.3604	2.191
Manufacture	0.0873	0.2057	2.356
Transportation Communication and Other Public Services	0.2049	0.4054	1.979
Trade	0.2064	0.3826	1.854
Finance, Insurance and Real Estate	0.1040	0.2725	2.620
Services	0.2548	0.4384	1.720
Government	0.5701	0.8580	1.505

Finance, Insurance, and Real Estate sector enjoys the highest income multiplier type 2 followed by the manufacturing sector. The Government sector has the lowest multiplier.

Employment multipliers

Local impact analyses are frequently preoccupied with the employment-creating effects of industrial expansion, because Commonwealth policymakers may be primarily and legitimately concerned with forecasting jobs in a particular area. For this reason, it is often useful to be able to derive employment multipliers as well as income multipliers from the I-O model.

The following method was used to estimate employment multipliers. Direct employment coefficients were defined as employment per million dollars of output. The vector of direct coefficients was multiplied by the local inverse in order to obtain the direct plus indirect coefficients.

Direct employment coefficients are shown in Table 6 below.

**TABLE 6
EMPLOYMENT, PRODUCTION, AND DIRECT EMPLOYMENT COEFFICIENTS, 1992**

Industrial Sector	Wages and salaries	Output (in million \$)	Direct Employment Coefficients*
Agriculture	34,000	701.6	48.46
Mining and Construction	55,000	3,357.2	16.38
Manufacture	164,000	31,748.4	5.17
Transportation Communication and Other Public Services	55,000	5,057.2	10.88
Trade	193,000	9,079.5	21.26
Finance, Insurance and Real Estate	32,000	8,177.2	3.91
Services	224,000	7,671.3	29.20
Government	219,000	6,292.8	34.80
Total	976,000	72,085.1	13.54

*Employment per million dollars of output

The results of the multiplication of direct coefficients vector (E) by the inverse matrix are the direct and indirect employment coefficients per unit of final demand (L).

Mathematically

$$L = E(I-A)^{-1}$$

where

E is the direct employment coefficients vector, L is the direct and indirect coefficients vector and, $(I-A)^{-1}$ is the inverse matrix..

Direct and indirect employment coefficients vector is divided by the direct coefficients to obtain the employment multipliers type 1. Employment multipliers type 1 are shown in table 7. As in the case of income multipliers, the employment multiplier of sector Finance, Insurance, and Real Estate is the highest. For economic planning purposes taking multipliers as a guide to employment creating policies may be misleading. Usually multipliers are very high when the direct employment coefficients are very low, as in the case of Finance sector (as well as for manufacturing).

**TABLE 7
EMPLOYMENT MULTIPLIERS TYPE I, PUERTO RICO, 1992**

Industrial Sector	Direct Employment Coefficients	Direct and Indirect Employment Coefficients	Type 1 Multipliers
Agriculture	48.46	53.24	1.099
Mining and Construction	16.38	25.50	1.556
Manufacture	5.17	11.01	2.131
Transportation Communication and Other Public Services	10.88	18.62	1.712
Trade	21.26	27.75	1.306
Finance, Insurance and Real Estate	3.91	11.78	3.011
Services	29.20	35.05	1.200
Government	34.80	40.37	1.160

Mathematically employment multipliers are obtained by using the following equations

Type 2 Employment Multipliers

$$K_{E1} = \frac{VectorL}{VectorE}$$

where

K_{E1} is the employment multiplier Type 1

The procedure to estimate the employment multiplier type 2 is the same as the one used to estimate income multiplier type 2. The vector of direct employment coefficients is

multiplied by the augmented inverse to obtain direct, indirect, and induced employment coefficients and the latter vector is divided by direct coefficients vector to get multipliers type 2. Mathematically

$$N = E(I-A)^{-1}$$

where N is the vector of direct, indirect and induced employment coefficients, E is the direct coefficient vector and $(I-A)^{-1}$ is the augmented inverse. The multiplier equation becomes

$$K_2 = N / E$$

where N and E are defined as before and K_2 are the employment multiplier type 2 shown in table 8 below.

**TABLE 8
EMPLOYMENT MULTIPLIERS TYPE 2, PUERTO RICO 1992**

Industrial Sector	Direct Employment Coefficients	Direct, Indirect and Induced Employment Coefficients	Employment Multipliers Type 2
Agriculture	48.46	62.06	1.280
Mining and Construction	16.38	33.4	2.040
Manufacture	5.17	15.52	3.000
Transportation Communication and Other Public Services	10.88	27.51	2.530
Trade	21.26	36.14	1.700
Finance, Insurance and Real Estate	3.91	17.76	4.540
Services	29.20	44.67	1.530
Government	34.80	59.19	1.700

Some Concluding Comments

It is difficult to offer firm generalizations about the size of I-O multipliers and their components in a regional economy, as is the case of Puerto Rico, because these can differ widely according to circumstances such as the industrial structure, the extent of interdependence among its sectors, its size and a lot of other factors. However, there are some general comments that can be made. For instance, the direct income change will tend to be higher in labor-intensive sectors, whereas capital-intensive industries with

strong links with other sectors in the regional economy may experience greater indirect effects. In the case of income multipliers, service industries tend to have high direct income effects because a substantial proportion of their costs consists of direct payments to factors of production (wages, rent, etc.) rather than purchase of materials. Moreover, the immediate leakage into imports tends to be much lower for service industries than for manufacturing. However, the tendency for indirect effects to be greater in manufacturing than service sectors while the converse holds for direct effects implies a narrowing in the range of values for the Type I multiplier. There is thus no clearly marked relationship between large direct income changes and large multipliers. It is very important to take account of induced income changes if we wish to attempt to capture the total income effect of changes in final demand. Type 2I multipliers tend to be considerably larger than Type I multipliers, and in most cases the induced income change is greater than the indirect income change.

As for the size of income multipliers, there are two safe generalizations. First, multiplier values vary widely from sector to sector, and the wider range highlights very clearly the importance of the sectoral composition of economic growth in raising regional incomes. Second, income multiplier values tend to vary directly with size of area; *ceteris paribus*, US multipliers are higher than local Commonwealth multipliers. The primary reason for this is that of the main leakages which reduce the generation of income. Imports tend to be the most important and certainly are the most variable with size of area. The propensity to import varies inversely with the scale of area; thus, large regions have high multipliers because they import less.

A subsidiary factor may be that the indirect income changes are greater in more complex regional economies, and we may reasonably expect the larger regions to have more interdependent economic structures.

Statistical Appendixes

The statistical appendixes include the 1992 revised multipliers at a disaggregated level (for 93 industrial sectors) and comparative multipliers for all the historical matrices so far

estimated for the Island. Persons interested in the disaggregated tables (in Excel) please request then to the author (angelruiz@prtc.net)